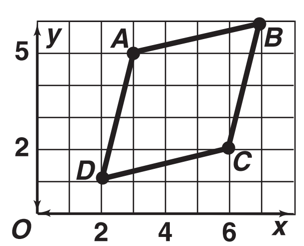
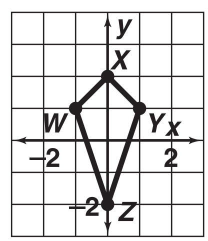
Answers for Lesson 6-1, pp. 308–311 Exercises

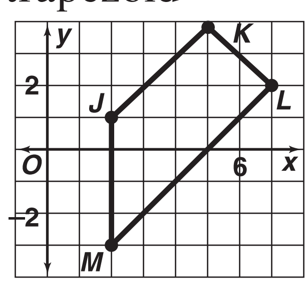
1. ▱, rectangle, rhombus, square
2. parallelogram
3. trapezoid
4. ▱, rhombus
5. kite
6. trapezoid, isosc. trapezoid
7. rhombus
8. parallelogram
9. rhombus
10. rectangle
11. kite
12. isosc. trapezoid
13. rhombus



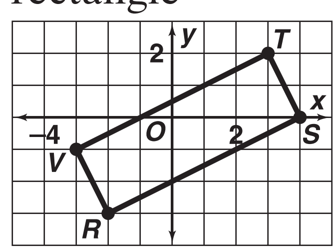
14. kite



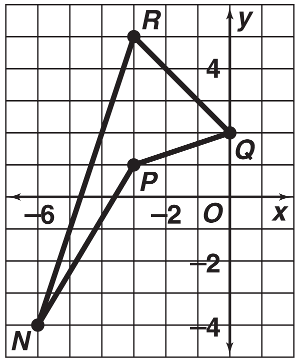
15. trapezoid



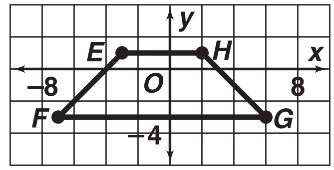
16. rectangle



17. quadrilateral



18. isos. trapezoid



19. $x = 11, y = 29$; 13, 13, 23, 23

20. $x = 4, y = 4.8$; 4.5, 4.5, 6.8, 6.8

21. $x = 2, y = 6$; 2, 7, 7, 2

22. $x = 1$; 4, 2, 4, 7

23. $x = 3, y = 5$; 15, 15, 15, 15

24. $x = 5, y = 4$; 3, 3, 3, 3

25. 40, 40, 140, 140; 11, 11, 15, 32

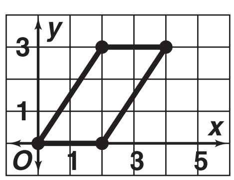
26. 58, 58, 122, 122; 6, 6, 6, 6

27. rectangle, square, trapezoid

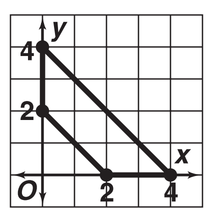
28. D

29–34. Answers may vary. Samples are given.

29.

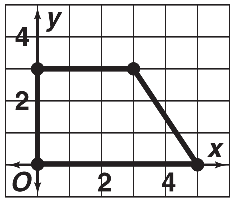


30.

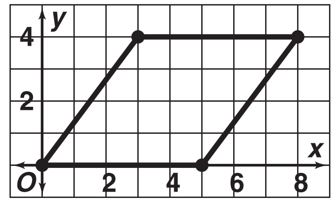


31. Impossible; a trapezoid with one rt. ∠ must have another, since two sides are ∥.

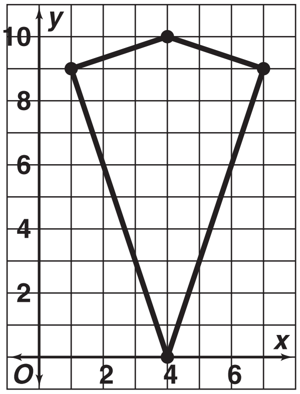
32.



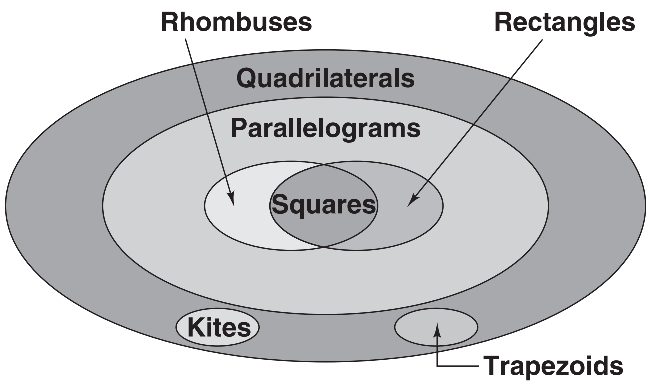
33.



34.



35.



36. True; a square is both a rectangle and a rhombus.

37. False; a trapezoid only has one pair of ∥ sides.

38. False; a kite does not have ≅ opp. sides.

39. True; all squares are ▱s.

40 False; kites are not ▱s.

41. False; only rhombuses with rt. ∠s are squares.

42. Rhombus; all 4 sides are ≅ because they come from the same cut.

43. Check students' work.

44. A rhombus has 4 ≅ sides, while a kite has 2 pairs of adj. sides ≅, but no opp. sides are ≅. Opp. sides of a rhombus are ∥, while opp. sides of a kite are not ∥.

45–48. Check students' sketches.

45. some isos. trapezoids, some trapezoids

46. ▱, rhombus, rectangle, square

47. rectangle, square

48. rhombus, square, kite, some trapezoids

49. A trapezoid has only one pair of ∥ sides.

50–53. Check students' sketches.

50. rectangle, ▱, kite

51. rhombus, ▱

52. square, rhombus, ▱

53. rhombus, ▱, kite

54–55. Check students' work.

56–59. Explanations may vary. Samples are given.

56. ▱, rectangle, trapezoid

57. ▱, kite, rhombus, trapezoid, isos. trapezoid

58. kite, ▱, rhombus, trapezoid, isos. trapezoid

59. ▱, rectangle, square, rhombus, kite, trapezoid

1. 127

2. 67

3. 76

4. 124

5. 100

6. 118

7. 3; 10, 20, 20

8. 22; 18.5, 23.6, 23.6

9. 20

10. 18

11. 17

12. 12; $m\angle Q = m\angle S = 36, m\angle P = m\angle R = 144$

13. 6; $m\angle H = m\angle J = 30, m\angle I = m\angle K = 150$

14. $x = 6, y = 8$

15. $x = 5, y = 7$

16. $x = 7, y = 10$

17. $x = 6, y = 9$

18. $x = 3, y = 4$

19. 12; 24

20. Pick 4 equally spaced lines on the paper. Place the paper so that the first button is on the first line and the last button is on the fourth line. Draw a line between the first and last buttons. The remaining buttons should be placed where the drawn line crosses the 2 $\parallel$ lines on the paper.

21. 3

22. 3

23. 6

24. 6

25. 9

26. 2.25

27. 2.25

28. 4.5

29. 4.5

30. 6.75

31. $BC = AD = 14.5$ in.; $AB = CD = 9.5$ in.

32. $BC = AD = 33$ cm; $AB = CD = 13$ cm

33. A

34. The opp. $\angle$s are $\cong$, so they have = measures. Consecutive $\angle$s are suppl., so their sum is 180.

35. **a.** $\overline{DC}$

b. $\overline{AD}$

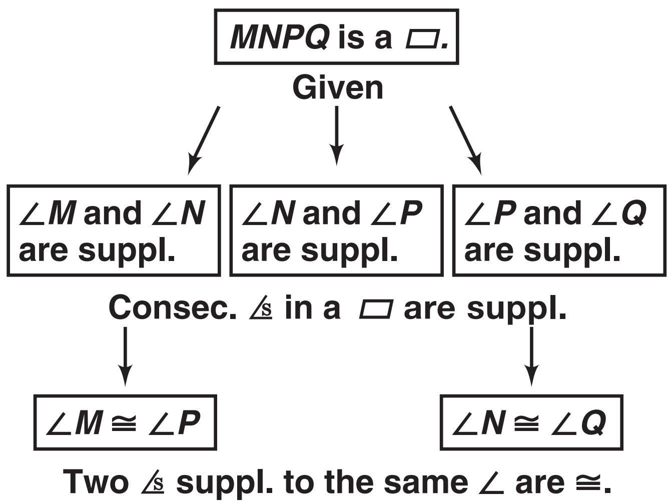
c. $\cong$

d. Reflexive

e. ASA

f. CPCTC

36.



37. 38, 32, 110 **38.** 81, 28, 71 **39.** 95, 37, 37

40. The lines going across may not be ∥ since they are not marked as ∥.

41. 18, 162

42. Answers may vary. Sample:

1. *LENS* and *NGTH* are ▱s. (Given)

2. $\angle ELS \cong \angle ENS$ and $\angle GTH \cong \angle GNH$ (Opp. ∠s of a ▱ are ≅.)

3. $\angle ENS \cong \angle GNH$ (Vertical ∠s are ≅.)

4. $\angle ELS \cong \angle GTH$ (Trans. Prop. of ≅)

43. Answers may vary. Sample: In ▱s *LENS* and *NGTH*, $\overline{GT} \parallel \overline{EH}$ and $\overline{EH} \parallel \overline{LS}$ by the def. of a ▱. Therefore $\overline{LS} \parallel \overline{GT}$ because if 2 lines are $\parallel$ to the same line then they are $\parallel$ to each other.

44. Answers may vary. Sample:

1. *LENS* and *NGTH* are ▱s. (Given)
2. $\angle GTH \cong \angle GNH$ (Opp. ∠s of a ▱ are $\cong$.)
3. $\angle ENS \cong \angle GNH$ (Vertical ∠s are $\cong$.)
4. $\angle LEN$ is supp. to $\angle ENS$ (Consec. ∠s in a ▱ are suppl.)
5. $\angle ENS \cong \angle GTH$ (Trans. Prop. of $\cong$)
6. $\angle E$ is suppl. to $\angle T$. (Suppl. of $\cong$ ∠s are suppl.)

45. $x = 12, y = 4$

46. $x = 0, y = 5$

47. $x = 9, y = 6$

48. Answers may vary. Sample: In ▱ *RSTW* and ▱ *XYTZ*, $\angle R \cong \angle T$ and $\angle X \cong \angle T$ because opp. ∠s of a ▱ are $\cong$. Then $\angle R \cong \angle X$ by the Trans. Prop. of $\cong$.

49. In ▱ *RSTW* and ▱ *XYTZ*, $\overline{XY} \parallel \overline{TW}$ and $\overline{RS} \parallel \overline{TW}$ by the def. of a ▱. Then $\overline{XY} \parallel \overline{RS}$ because if 2 lines are $\parallel$ to the same line, then they are $\parallel$ to each other.

50. $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$ by def. of ▱. $\angle 2 \cong \angle 3$ and $\angle 1 \cong \angle 4$ by alt. int. ∠s. $\angle 1 \cong \angle 2$ by def. of $\angle$ bisect., so $\angle 3 \cong \angle 4$ by Trans. Prop. of $\cong$.

51. **a.** Answers may vary. Check students' work.

b. No; the corr. sides can be $\cong$ but the ∠s may not be.

52. a. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$ and $\overline{AC} \cong \overline{CE}$ (Given)

b. $ABGC$ and $CDHE$ are parallelograms. (Def. of a ▱)

c. $\overline{BG} \cong \overline{AC}$ and $\overline{DH} \cong \overline{CE}$ (Opp. sides of a ▱ are $\cong$.)

d. $\overline{BG} \cong \overline{DH}$ (Trans. Prop. of $\cong$)

e. $\overline{BG} \parallel \overline{DH}$ (If 2 lines are $\parallel$ to the same line, then they are $\parallel$ to each other.)

f. $\angle 2 \cong \angle 1$, $\angle 1 \cong \angle 4$, $\angle 4 \cong \angle 5$, and $\angle 3 \cong \angle 6$ (If 2 lines are $\parallel$, then the corr. ∠s are $\cong$.)

g. $\angle 2 \cong \angle 5$ (Trans. Prop. of $\cong$)

h. $\triangle BGD \cong \triangle DHF$ (AAS)

i. $\overline{BD} \cong \overline{DF}$ (CPCTC)

53. a. Given: 2 sides and the included ∠ of ▱$ABCD$ are $\cong$ to the corr. parts of ▱$WXYZ$. Let $\angle A \cong \angle W$, $\overline{AB} \cong \overline{WX}$ and $\overline{AD} \cong \overline{WZ}$. Since opp. ∠s of a ▱ are $\cong$, $\angle A \cong \angle C$ and $\angle W \cong \angle Y$. Thus $\angle C \cong \angle Y$ by the Trans. Prop. of $\cong$. Similarly, opp. sides of a ▱ are $\cong$, thus $\overline{AB} \cong \overline{CD}$ and $\overline{WX} \cong \overline{ZY}$. Using the Trans. Prop. of $\cong$, $\overline{CD} \cong \overline{ZY}$. The same can be done to prove $\overline{BC} \cong \overline{XY}$. Since consec. ∠s of a ▱ are suppl., $\angle A$ is suppl. to $\angle D$, and $\angle W$ is suppl. to $\angle Z$. Suppls. of $\cong$ ∠s are $\cong$, thus $\angle D \cong \angle Z$. The same can be done to prove $\angle B \cong \angle X$. Therefore, since all corr. ∠s and sides are $\cong$, ▱$ABCD \cong$ ▱$WXYZ$.

b. No; opp. ∠s and sides are not necessarily $\cong$ in a trapezoid.

Answers for Lesson 6-3, pp. 324–326 Exercises

1. 5 **2.** $x = 3, y = 4$ **3.** $x = 1.6, y = 1$

4. $\frac{5}{3}$ **5.** 5 **6.** 13

7. Yes; both pairs of opp. sides are ≅.

8. No; the quad. could be a kite.

9. Yes; both pairs of opp. ∠s are ≅.

10. It remains a ▱ because the shelves and connecting pieces remain ∥.

11. A quad. is a ▱ if and only if opp. sides are ≅ (6-1 and 6-5); opp. ∠s are ≅ (6-2 and 6-6); diags. bis. each other (6-3 and 6-7).

12. **a.** Distr. Prop.

b. Div. Prop. of Eq.

c. $\overline{AD} \parallel \overline{BC}, \overline{AB} \parallel \overline{DC}$

d. If same-side int. ∠s are suppl., the lines are ∥.

e. Def. of ▱

13. Draw diagonals $\overline{TX}$ and $\overline{WY}$ intersecting at R.

a. $\overline{TW} \cong \overline{YX}$ (Given)

b. $\angle TWR \cong \angle XYR$ (Alt. Int. ∠s ≅)

c. $\angle WTR \cong \angle YXR$ (Alt. Int. ∠s ≅)

d. $\triangle TWR \cong \triangle YXR$ (ASA)

e. $\overline{WR} \cong \overline{YR}$ (CPCTC)

f. $\overline{TR} \cong \overline{XR}$ (CPCTC)

g. The diagonals bisect each other. (def. of bis.)

h. $TWXY$ is a ▱ (Thm. 6-7).

14. $x = 15, y = 25$

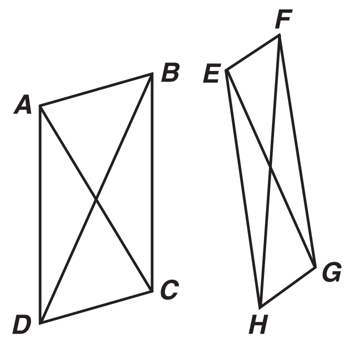
15. $x = 3, y = 11$

16. $c = 8, a = 24$

17. $k = 9, m = 23.4$

18. D

19. Answers may vary. Sample:



20. $\angle JKN \cong \angle LMN$ (given), $\angle LKN \cong \angle JMN$ (given), and $\overline{MK} \cong \overline{MK}$, so $\triangle JKM \cong \triangle LMK$ by ASA. $\overline{JK} \cong \overline{ML}$ and $\overline{MJ} \cong \overline{LK}$ (CPCTC), so $JKLM$ is a ▱ because opp. sides are $\cong$ (Thm. 6-5).

21. $\triangle TRS \cong \triangle RTW$ (given), so $\overline{ST} \cong \overline{RW}$ and $\overline{SR} \cong \overline{TW}$. $RSTW$ is a ▱ because opp. sides are $\cong$ (Thm. 6-5).

22. (4, 0)

23. (6, 6)

24. (−2, 4)

25. You can show a quad. is a ▱ if both pairs of opp. sides are $\parallel$ or $\cong$, if both pairs of opp. ∠s are $\cong$, if diagonals bisect each other, if all consecutive ∠s are suppl., or if one pair of opp. sides is both $\parallel$ and $\cong$.

26. $\frac{1}{6}$

27. Answers may vary. Sample:

1. $\overline{AB} \cong \overline{CD}$, $\overline{AC} \cong \overline{BD}$ (Given)
2. $ACDB$ is a ▱. (If opp. sides of a quad. are ≅, then it is a ▱.)
3. M is the midpoint of $\overline{BC}$. (The diag. of a ▱ bisect each other.)
4. $\overline{AM}$ is a median. (Def. of a median)

28. $G(-4, 1)$, $H(1, 3)$

Answers for Lesson 6-4, pp. 332–335 Exercises

1. 38, 38, 38, 38

2. 26, 128, 128

3. 118, 31, 31

4. 33.5, 33.5, 113, 33.5

5. 32, 90, 58, 32

6. 90, 60, 60, 30

7. 55, 35, 55, 90

8. 60, 90, 30

9. 90, 55, 90

10. 4; $LN = MP = 4$

11. 3; $LN = MP = 7$

12. 1; $LN = MP = 4$

13. 9; $LN = MP = 67$

14. $\frac{5}{3}$; $LN = MP = \frac{29}{3} = 9\frac{2}{3}$

15. $\frac{5}{2}$; $LN = MP = 12\frac{1}{2}$

16. rhombus; one diag. bis. 2 ∠s of the ▱ (Thm. 6-12).

17. rhombus; the diags. are $\perp$.

18. neither; the figure could be a ▱ that is neither a rhombus nor a rect.

19. The pairs of opp. sides of the frame remain $\cong$, so the frame remains a ▱.

20. After measuring the sides, she can measure the diagonals. If the diags. are $\cong$, then the figure is a rectangle by Thm. 6-14.

21. Square; a square is both a rectangle and a rhombus, so its diag. have the properties of both.

22. **a.** Def. of a rhombus

b. Diagonals of a ▱ bisect each other.

c. $\overline{AE} \cong \overline{AE}$

d. Reflexive Prop. of $\cong$

e. $\triangle ABE \cong \triangle ADE$

f. CPCTC

g. $\angle$ Add. Post.

h. $\angle AEB$ and $\angle AED$ are rt. $\angle$s.

i. $\cong$ suppl. $\angle$s are rt. $\angle$s Thm.

j. Def. of $\perp$

23. Answers may vary. Sample: The diagonals of a ▱ bisect each other so $\overline{AE} \cong \overline{CE}$. Both $\angle AED$ and $\angle CED$ are right $\angle$s because $\overline{AC} \perp \overline{BD}$, and since $\overline{DE} \cong \overline{DE}$ by the Reflexive Prop., $\triangle AED \cong \triangle CED$ by SAS. By CPCTC $\overline{AD} \cong \overline{CD}$, and since opp. sides of a ▱ are $\cong$, $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$. So $ABCD$ is a rhombus because it has 4 $\cong$ sides.

24. A

25–34. **Symbols may vary. Samples are given:**
parallelogram: ▱
rhombus: 🅁
rectangle: ▭
square: 🅂

25. 🅁, 🅂

26. ▱, 🅁, ▭, 🅂

27. ▱, 🅁, ▭, 🅂

28. ▱, 🅁, ▭, 🅂

29. ▭, 🅂

30. ▱, 🅁, ▭, 🅂

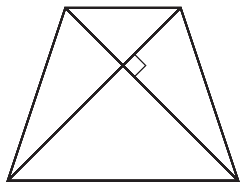
31. ▱, 🅁, ▭, 🅂

32. ▭, 🅂

33. ▱R, ▢S

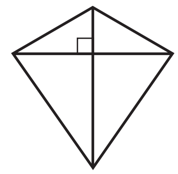
34. ▱R, ▢S

35.



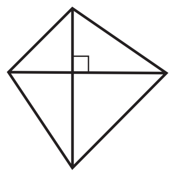
Diag. are ≅, diag. are ⊥.

36.



Diag. are ⊥ and ≅.

37.



Diag. are ≅, diag. are ⊥.

38. **a.** Opp. sides are ≅ and ∥; diag. bis. each other; opp. $\angle$s are ≅; consec. $\angle$s are suppl.

b. All sides are ≅; diag. are ≅.

c. All $\angle$s are rt. $\angle$s; diag. are ⊥ bis. of each other; each diag. bis. two $\angle$s.

39. **1.** $ABCD$ is a parallelogram. (Given)
$\overline{AC}$ bisects $\angle BAD$ and $\angle BCD$. (Given)

2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$ (Def. of bisect)

3. $\overline{AC} \cong \overline{AC}$ (Refl. Prop. of ≅)

4. $\triangle ABC \cong \triangle ADC$ (ASA)

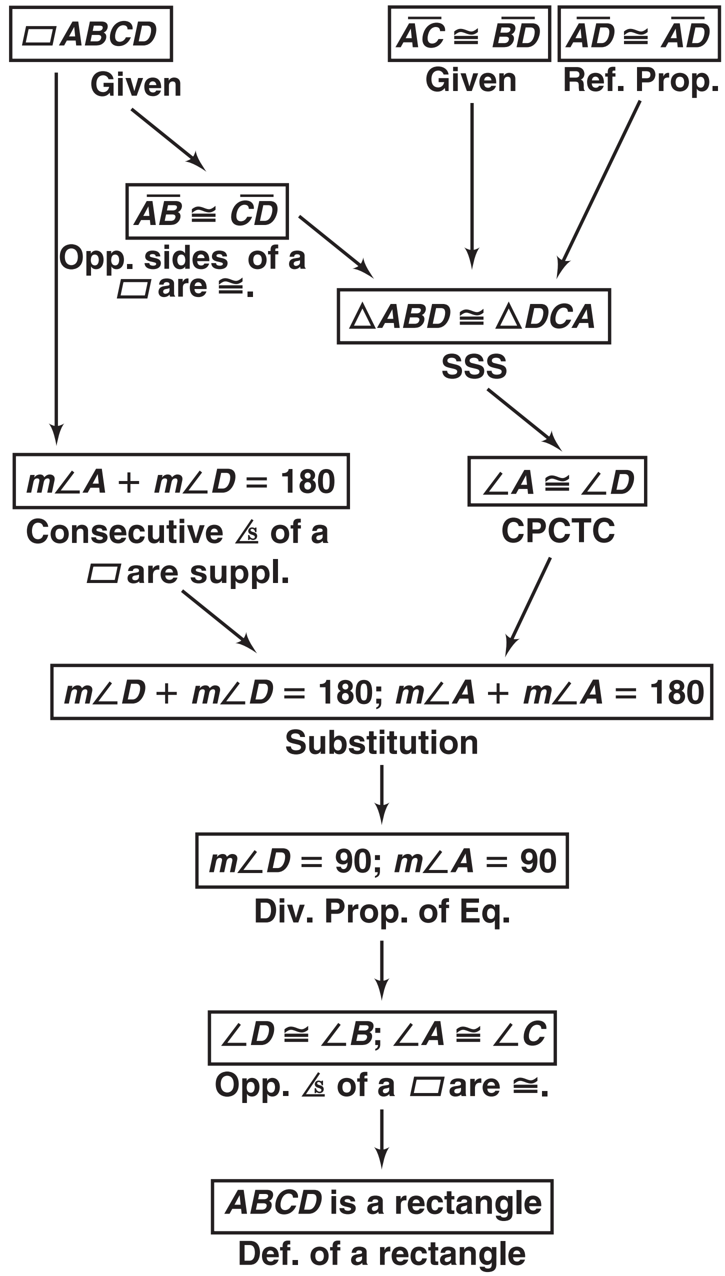
5. $\overline{AB} \cong \overline{AD}$ (CPCTC)

6. $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$ (Opp. sides of a ▱ are ≅.)

7. $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ (Trans. Prop. of ≅)

8. $ABCD$ is a rhombus. (Def. of rhomb.)

40.



41. Yes; since all right ∠s are ≅, the opp. ∠s are ≅ and it is a ▱. Since it has all right ∠s, it is a rectangle.

42. Yes; 4 sides are ≅, so the opp. sides are ≅ making it a ▱. Since it has 4 ≅ sides it is also a rhombus.

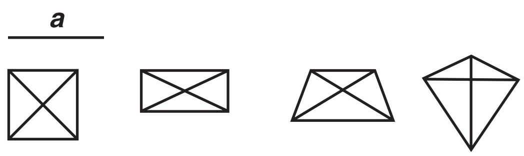
43. Yes; a quad. with 4 ≅ sides is a ▱ and a ▱ with 4 ≅ sides and 4 right ∠s is a square.

44. 30

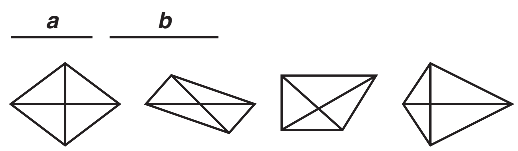
45. $x = 5, y = 32, z = 7.5$ **46.** $x = 7.5, y = 3$

47–49. Drawings may vary. Samples are given.

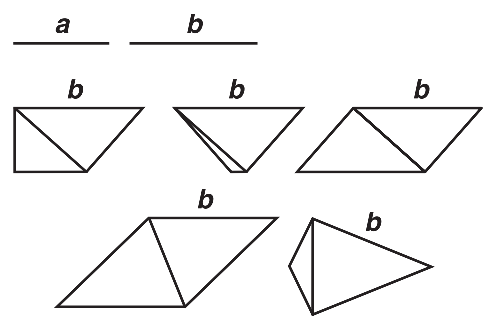
47. Square, rectangle, isosceles trapezoid, kite



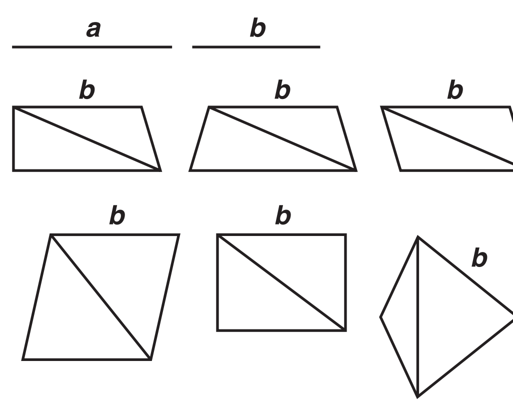
48. Rhombus, ▱, trapezoid, kite



49. For $a < b$: trapezoid, isosc. trapezoid $\left(a > \frac{1}{2}b\right)$, ▱, rhombus, kite



For $a > b$: trapezoid, isosc. trapezoid, ▱, rhombus $(a < 2b)$, kite, rectangle, square $\left(\text{if } a = \sqrt{2}b\right)$



50. 16, 16 **51.** 2, 2 **52.** 1, 1 **53.** 1, 1

54–59. Answers may vary. Samples are given.

54. Draw diag. 1, and construct its midpt. Draw a line through the mdpt. Construct segments of length diag. 2 in opp. directions from mdpt. Then, bisect these segments. Connect these mdpts. with the endpts. of diag. 1.

55. Construct a rt. ∠, and draw diag. 1 from its vertex. Construct the ⊥ from the opp. end of diag. 1 to a side of the rt. ∠. Repeat to other side.

56. Same as 54, but construct a ⊥ line at the midpt. of diag. 1.

57. Same as 56, except make the diag. ≅.

58. Draw diag. 1. Construct a ⊥ at a pt. different than the mdpt. Construct segments on the ⊥ line of length diag. 2 in opp. directions from the pt. Then, bisect these segments. Connect these midpts. to the endpts. of diag. 1.

59. Draw an acute ∠. Use the compass to mark the length of diag. 1 on one side of the angle. The other side will be a base for the trap. Construct a line ∥ to the base through the non-vertex endpt. of diag. 1. Set the compass to the length of diag. 2 and place the point on the non-vertex endpt. of the base. Draw an arc that intersects the line ∥ to the base. Draw diag. 2 through these two points. Finish by drawing the non-∥ sides of the trap.

60. Impossible; if the diag. of a ▱ are ≅, then it would have to be a rectangle and have right ∠s.

61. Yes; ≅ diag. in a ▱ mean it can be a rectangle with 2 opp. sides 2 cm long.

62. Impossible; in a ▱, consecutive ∠s must be supp., so all ∠s must be right ∠s. This would make it a rectangle.

63. Given ▱$ABCD$ with diag. $\overline{AC}$. Let $\overline{AC}$ bisect $\angle BAD$. Because $\triangle ABC \cong \triangle DAC$, $AB = DA$ by CPCTC. But since opp. sides of a ▱ are ≅, $AB = CD$ and $BC = DA$. So $AB = BC = CD = DA$, and ▱$ABCD$ is a rhombus. The new statement is true.

1. 77, 103, 103

2. 69, 69, 111

3. 49, 131, 131

4. 105, 75, 75

5. 115, 115, 65

6. 120, 120, 60

7. **a.** isosc. trapezoids

b. 69, 69, 111, 111

8. 90, 68

9. 90, 45, 45

10. 108, 108

11. 90, 26, 90

12. 90, 40, 90

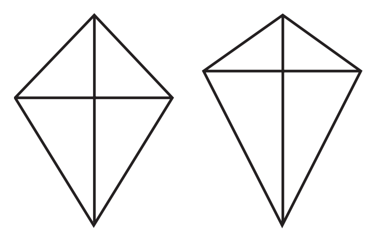
13. 90, 55, 90, 55, 35

14. 90, 52, 38, 37, 53

15. 90, 90, 90, 90, 46, 34, 56, 44, 56, 44

16. 112, 112

17. Answers may vary. Sample:



18. 12, 12, 21, 21

19. Explanations may vary. Sample: If both ∠s are bisected, then this combined with $\overline{KM} \cong \overline{KM}$ by the Reflexive Prop. means $\triangle KLM \cong \triangle KNM$ by SAS. By CPCTC, $\angle L \cong \angle N$. $\angle L$ and $\angle N$ are opp. ∠s, but if $KLMN$ is isos., both pairs of base ∠s are also ≅. By the Trans. Prop., all 4 angles are ≅, so $KLMN$ must be a rect. or a square. This contradicts what is given, so $\overline{KM}$ cannot bisect $\angle LMN$ and $\angle LKN$.

20. 12

21. 15

22. 15

23. 3

24. 4

25. 1

26. 28

27. $x = 35, y = 30$

28. $x = 18, y = 108$

29. Isosc. trapezoid; all the large rt. △s appear to be ≅.

30. 112, 68, 68

31. Yes, the ≅ ∠s can be obtuse.

32. Yes, the ≅ ∠s can be obtuse, as well as one other ∠.

33. Yes; if 2 ≅ ∠s are rt. ∠s, they are suppl. The other 2 ∠s are also suppl.

34. No; if two consecutive ∠s are suppl., then another pair must be also because one pair of opp. ∠s is ≅. Therefore, the opp. ∠s would be ≅, which means the figure would be a ▱ and not a kite.

35. Yes; the ≅ ∠s may be 45° each.

36. No; if two consecutive ∠s were compl., then the kite would be concave.

37. Rhombuses and squares would be kites since opp. sides can be ≅ also.

38. **1.** $ABCD$ is an isos. trapezoid, $\overline{AB} \cong \overline{DC}$. (Given)

2. Draw $\overline{AE} \parallel \overline{DC}$. (Two points determine a line.)

3. $\overline{AD} \parallel \overline{EC}$ (Def. of a trapezoid)

4. $AECD$ is a ▱. (Def. of a ▱)

5. $\angle C \cong \angle 1$ (Corr. ∠s are ≅.)

6. $\overline{DC} \cong \overline{AE}$ (Opp. sides of a ▱ are ≅.)

7. $\overline{AB} \cong \overline{AE}$ (Trans. Prop. of ≅)

8. $\triangle AEB$ is an isosc. △. (Def. of an isosc. △)

9. $\angle B \cong \angle 1$ (Base ∠s of an isosc. △ are ≅.)

10. $\angle B \cong \angle C$ (Trans. Prop. of ≅)

11. $\angle B$ and $\angle BAD$ are suppl., $\angle C$ and $\angle CDA$ are suppl. (Same side int. ∠s are suppl.)

12. $\angle BAD \cong \angle CDA$ (Suppl. of ≅ ∠s are ≅.)

39. Answers may vary. Sample: Draw $\overline{TA}$ and $\overline{RP}$.

1. isosc. trapezoid $TRAP$ (Given)

2. $\overline{TA} \cong \overline{PR}$ (Diag. of an isosc. trap. are ≅.)

3. $\overline{TR} \cong \overline{PA}$ (Given)

4. $\overline{RA} \cong \overline{RA}$ (Refl. Prop. of ≅)

5. $\triangle TRA \cong \triangle PAR$ (SSS)

6. $\angle RTA \cong \angle APR$ (CPCTC)

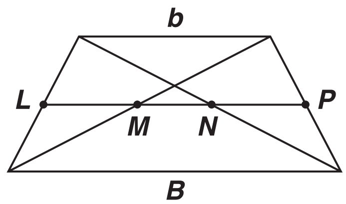
40. Draw $\overline{BI}$ as described, then draw $\overline{BT}$ and $\overline{BP}$.

1. $\overline{TR} \cong \overline{PA}$ (Given)
2. $\angle R \cong \angle A$ (Base ∠s of isosc. trap. are ≅.)
3. $\overline{RB} \cong \overline{AB}$ (Def. of bisector)
4. $\triangle TRB \cong \triangle PAB$ (SAS)
5. $\overline{BT} \cong \overline{BP}$ (CPCTC)
6. $\angle RBT \cong \angle ABP$ (CPCTC)
7. $\angle TBI \cong \triangle PBI$ (Compl. of ≅ ∠s are ≅.)
8. $\overline{BI} \cong \overline{BI}$ (Refl. Prop. of ≅)
9. $\triangle TBI \cong \triangle PBI$ (SAS)
10. $\angle BIT \cong \angle BIP$ (CPCTC)
11. $\angle BIT$ and $\angle BIP$ are rt. ∠s. (≅ suppl. ∠s are rt. ∠s.)
12. $\overline{TI} \cong \overline{PI}$ (CPCTC)
13. $\overline{BI}$ is ⊥ bis. of $\overline{TP}$. (Def. of ⊥ bis.)

41–42. Check students' justifications. Samples are given.

41. It is one half the sum of the lengths of the bases; draw a diag. of the trap. to form 2 △s. The bases B and b of the trap. are each a base of a △. Then the segment joining the midpts. of the non-∥ sides is the sum of the midsegments of the △s. This sum is $\frac{1}{2}B + \frac{1}{2}b = \frac{1}{2}(B + b)$.

42. It is one half the difference of the lengths of the bases. By the $\triangle$ Midsegment Thm. and the $\|$ Post., midpoints L, M, N, and P are collinear. $MN = LN - LM = \frac{1}{2}B - \frac{1}{2}b$ ($\triangle$ Midsegment Thm.) $= \frac{1}{2}(B - b)$.



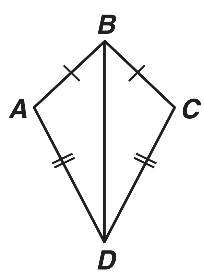
43. D is any point on $\overleftrightarrow{BN}$ such that $ND \neq BN$ and D is below N.

44. **1.** $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$ (Given)

2. $\overline{BD} \cong \overline{BD}$ (Refl. Prop. of $\cong$)

3. $\triangle ABD \cong \triangle CBD$ (SSS)

4. $\angle A \cong \angle C$ (CPCTC)



1. **a.** $(2a, 0)$

b. $(0, 2b)$

c. (a, b)

d. $\sqrt{b^2 + a^2}$

e. $\sqrt{b^2 + a^2}$

f. $\sqrt{b^2 + a^2}$

g. $MA = MB = MC$

2. $W(0, h); Z(b, 0)$

3. $W(a, a); Z(a, 0)$

4. $W(-b, b); Z(-b, -b)$

5. $W(0, b); Z(a, 0)$

6. $W(-r, 0); Z(0, -t)$

7. $W(-b, c); Z(0, c)$

8. Answers may vary. Sample: $r = 3, t = 2$; slopes are $\frac{2}{3}$ and $-\frac{2}{3}$; all lengths are $\sqrt{13}$; the opp. sides have the same slope, so they are $\parallel$. The 4 sides are $\cong$.

9. **a.** Diag. of a rhombus are $\perp$.

b. Diag. of a ▱ that is not a rhombus are not $\perp$.

10–15. Answers may vary. Samples are given.

10. A, C, H, F

11. B, D, H, F

12. A, B, F, E

13. A, C, G, E

14. A, C, F, E

15. A, D, G, F

16. $W(0, 2h); Z(2b, 0)$

17. $W(2a, 2a); Z(2a, 0)$

18. $W(-2b, 2b); Z(-2b, -2b)$

19. $W(0, b); Z(2a, 0)$

20. $W(-2r, 0); Z(0, -2t)$

21. $W(-2b, 2c); Z(0, 2c)$

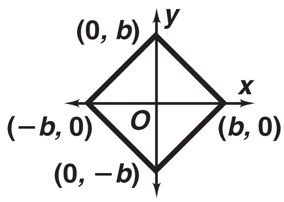
22. A

23. $(c - a, b)$

24. $(a, 0)$

35. $(-b, 0)$

26. **a.**



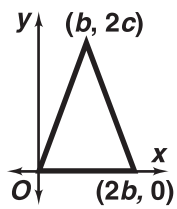
b. $(-b, 0), (0, b), (b, 0), (0, -b)$

c. $b\sqrt{2}$

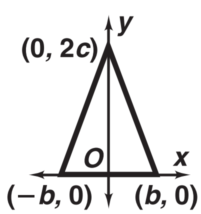
d. $1, -1$

e. Yes, because the product of the slopes is -1.

27 **a.**



b.

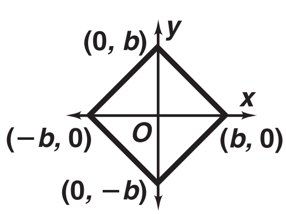


c. $\sqrt{b^2 + 4c^2}$

d. $\sqrt{b^2 + 4c^2}$

e. The lengths are $=$.

28.



29. Step 1: $(0, 0)$

Step 2: $(a, 0)$

Step 3: Since $m\angle 1 + m\angle 2 + 90 = 180$, $\angle 1$ and $\angle 2$ must be compl. $\angle 3$ and $\angle 2$ are the acute ∠s of a rt. △.

Step 4: $(-b, 0)$

Step 5: $(-b, a)$

Step 6: Using the formula for slope, the slope for $\ell_1 = \frac{b}{a}$ and the slope for $\ell_2 = -\frac{a}{b}$. Mult. the slopes, $\frac{b}{a} \cdot -\frac{a}{b} = -1$.

Answers for Lesson 6-7, pp. 349–353 Exercises

1. a. $W\left(\frac{a}{2}, \frac{b}{2}\right); Z\left(\frac{c+e}{2}, \frac{d}{2}\right)$

b. $W(a, b); Z(c + e, d)$

c. $W(2a, 2b); Z(2c + 2e, 2d)$

d. c; it uses multiples of 2 to name the coordinates of W and Z.

2. a. origin

b. x-axis

c. 2

d. coordinates

3. a. y-axis

b. Distance

4. a. rt. ∠

b. legs

c. multiples of 2

d. M

e. N

f. Midpoint

g. Distance

5. a. isos.

b. x-axis

c. y-axis

d. midpts.

e. ≅ sides

f. slopes

g. the Distance Formula

6. **a.** $\sqrt{(b + a)^2 + c^2}$

b. $\sqrt{(a + b)^2 + c^2}$

7. **a.** $\sqrt{a^2 + b^2}$

b. $2\sqrt{a^2 + b^2}$

8. **a.** $D(-a - b, c), E(0, 2c), F(a + b, c), G(0, 0)$

b. $\sqrt{(a + b)^2 + c^2}$

c. $\sqrt{(a + b)^2 + c^2}$

d. $\sqrt{(a + b)^2 + c^2}$

e. $\sqrt{(a + b)^2 + c^2}$

f. $\frac{c}{a + b}$

g. $\frac{c}{a + b}$

h. $-\frac{c}{a + b}$

i. $-\frac{c}{a + b}$

j. sides

k. $DEFG$

9. **a.** (a, b)

b. (a, b)

c. the same point

10. Answers may vary. Sample: The △ Midsegment Thm.; the segment connecting the midpts. of 2 sides of the △ is ∥ to the 3rd side and half its length; you can use the Midpoint Formula and the Distance Formula to prove the statement directly.

11. The vertices of $KLMN$ are $L(b, a + c)$, $M(b, c)$, $N(-b, c)$, and $K(-b, a + c)$. The slopes of $\overline{KL}$ and $\overline{MN}$ are zero, so these segments are horizontal. The endpoints of $\overline{KN}$ have equal x-coordinates and so do the endpoints of $\overline{LM}$. So these segments are vertical. Hence opposite sides of $KLMN$ are parallel and consecutive sides are $\perp$. It follows that $KLMN$ is a rectangle.

12–23. Answers may vary. Samples are given.

12. yes; Dist. Formula

13. yes; same slope

14. yes; prod. of slopes $= -1$

15. no; may not have intersection pt.

16. no; may need $\angle$ measures

17. no; may need $\angle$ measures

18. yes; prod. of slopes of sides of $\angle A = -1$

19. yes; Dist. Formula

20. yes; Dist. Formula, 2 sides $=$

21. no; may need $\angle$ measures

22. yes; intersection pt. for all 3 segments

23. yes; Dist. Formula, $AB = BC = CD = AD$

24. A

25. 1, 4, 7

26. 0, 2, 4, 6, 8

27. −0.8, 0.4, 1.6, 2.8, 4, 5.2, 6.4, 7.6, 8.8

28. −1.76, −1.52, −1.28, . . . , 9.52, 9.76

29. $-2 + \frac{12}{n}, -2 + 2\left(\frac{12}{n}\right), -2 + 3\left(\frac{12}{n}\right), \ldots, -2 + (n-1)\left(\frac{12}{n}\right)$

30. (0, 7.5), (3, 10), (6, 12.5)

31. $\left(-1, 6\frac{2}{3}\right), \left(1, 8\frac{1}{3}\right), (3, 10), \left(5, 11\frac{2}{3}\right), \left(7, 13\frac{1}{3}\right)$

32. (−1.8, 6), (−0.6, 7), (0.6, 8), (1.8, 9), (3, 10), (4.2, 11), (5.4, 12), (6.6, 13), (7.8, 14)

33. (−2.76, 5.2), (−2.52, 5.4), (−2.28, 5.6), . . . , (8.52, 14.6), (8.76, 14.8)

34. $\left(-3 + \frac{12}{n}, 5 + \frac{10}{n}\right), \left(-3 + 2\left(\frac{12}{n}\right), 5 + 2\left(\frac{10}{n}\right)\right), \ldots,$
$\left(-3 + (n-1)\left(\frac{12}{n}\right), 5 + (n-1)\left(\frac{10}{n}\right)\right)$

35. **a.** $L(b, d), M(b + c, d), N(c, 0)$

b. $\overleftrightarrow{AM}: y = \frac{d}{b+c}x$; $\overleftrightarrow{BN}: y = \frac{2d}{2b-c}(x - c)$;
$\overleftrightarrow{CL}: y = \frac{d}{b-2c}(x - 2c)$

c. $P\left(\frac{2(b+c)}{3}, \frac{2d}{3}\right)$

d. Pt. P satisfies the eqs. for $\overleftrightarrow{AM}$ and $\overleftrightarrow{CL}$.

e. $AM = \sqrt{(b+c)^2 + d^2}$; $AP = \sqrt{\left(\frac{2(b+c)}{3}\right)^2 + \left(\frac{2d}{3}\right)^2} =$
$\sqrt{\left(\frac{2}{3}\right)^2\left((b+c)^2 + d^2\right)} = \frac{2}{3}\sqrt{(b+c)^2 + d^2} = \frac{2}{3}AM$

The other 2 distances are found similarly.

36. **a.** $\frac{b}{c}$

b. Let a pt. on line p be (x, y). Then the eq. of p is $\frac{y - 0}{x - a} = \frac{b}{c}$ or $y = \frac{b}{c}(x - a)$.

c. $x = 0$

d. When $x = 0$, $y = \frac{b}{c}(x - a) = \frac{b}{c}(-a) = -\frac{ab}{c}$. So p and q intersect at $\left(0, -\frac{ab}{c}\right)$.

e. $\frac{a}{c}$

f. Let a pt. on line r be (x, y). Then the eq. of r is $\frac{y - 0}{x - b} = \frac{a}{c}$ or $y = \frac{a}{c}(x - b)$.

g. $-\frac{ab}{c} = \frac{a}{c}(0 - b)$

h. $\left(0, -\frac{ab}{c}\right)$

37. Assume $b > a$. $a + \frac{b - a}{n}, a + 2\left(\frac{b - a}{n}\right), \ldots,$
$a + (n - 1)\left(\frac{b - a}{n}\right)$

38. Assume $b \geq a, d \geq c$. $\left(a + \frac{b - a}{n}, c + \frac{d - c}{n}\right),$
$\left(a + 2\left(\frac{b - a}{n}\right), c + 2\left(\frac{d - c}{n}\right)\right), \ldots,$
$\left(a + (n - 1)\left(\frac{b - a}{n}\right), c + (n - 1)\left(\frac{d - c}{n}\right)\right)$

39. **a.** The △s with bases d and b, and heights c and a, respectively, have the same area. They share the small right △ with base d and height c, and the remaining areas are △s with base c and height $(b - d)$. So $\frac{1}{2}ad = \frac{1}{2}bc$. Mult. both sides by 2 gives $ad = bc$.

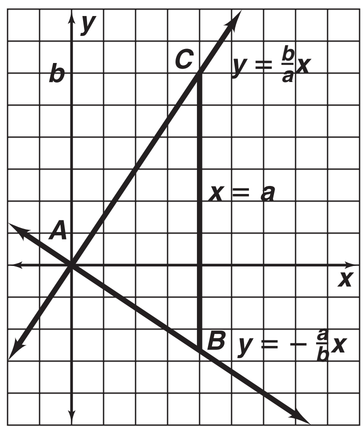
b. The diagram shows that $\frac{a}{b} = \frac{c}{d}$, since both represent the slope of the top segment of the △. So by (a), $ad = bc$.

40. Divide the quad. into 2 ⚠. Find the centroid for each △ and connect them. Now divide the quad. into 2 other ⚠ and follow the same steps. Where the two lines meet connecting the centroids of the 4 ⚠ is the centroid of the quad.

41. **a.** Horiz. lines have slope 0, and vert. lines have undef. slope. Neither could be mult. to get -1.

b. Assume the lines do not intersect. Then they have the same slope, say m. Then $m \cdot m = m^2 = -1$, which is impossible. So the lines must intersect.

c. Let the eq. for ℓ_1 be $y = \frac{b}{a}x$, and for ℓ_2 be $y = -\frac{a}{b}x$, and the origin be the int. point.



Define $C(a, b)$, $A(0, 0)$, and $B\left(a, -\frac{a^2}{b}\right)$. Using the Distance Formula, $AC = \sqrt{a^2 + b^2}$, $BA = \sqrt{a^2 + \frac{a^4}{b^2}}$, and $CB = b + \frac{a^2}{b}$. Then $AC^2 + BA^2 = CB^2$, and $m\angle A = 90$ by the Conv. of the Pythagorean Thm. So $\ell_1 \perp \ell_2$.