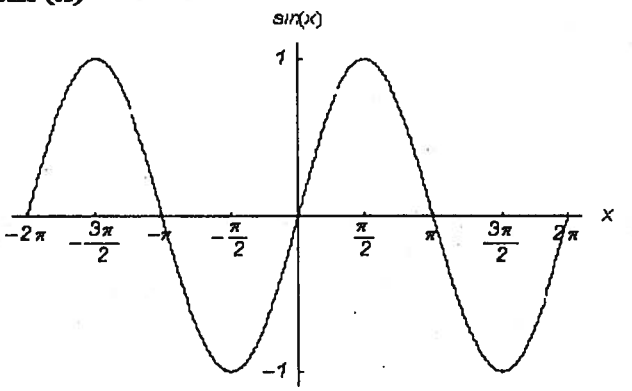
Lesson 13.7: Graphing Translations of Trigonometric Functions

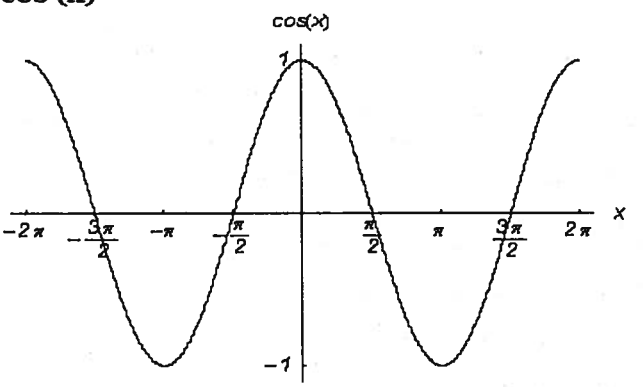
sin (x)



sin(x)

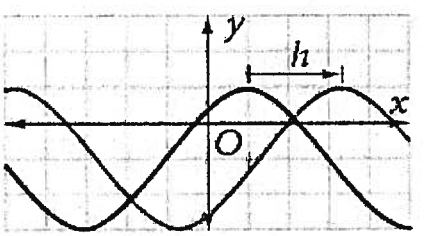
Domain: all real numbers
Range: $-1 \le y \le 1$
Amplitude: 1
Period: 2π

cos (x)

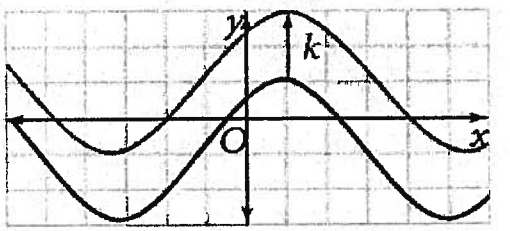


cos(x)
Domain: all real numbers
Range: $-1 \le y \le 1$
Amplitude: 1
Period: 2π

You can translate periodic functions horizontally and vertically using the methods we have used for other functions. A horizontal translation of a periodic function is a phase shift.



$g(x)$: horizontal translation of $f(x)$
$g(x) = f(x - h)$



$h(x)$: vertical translation of $f(x)$
$h(x) = f(x) + k$

When *g(x) = f(x – h)*, the value of *h* is the amount of the shift left or right. If h > 0, the shift is to the right & if h < 0, the shift it to the left.

When g(x) = f(x) + k, the value of k is the amount of the shift up or down. If k > 0, then the shift is up & if k < 0, the shift is down.

Example:

Describe the phase shift for each translation below.

a. *g(x) = f(x – 2)* $h=2$

Shift 2 units right

b. *y = cos(x + 4)* $h=-4$

Shift 4 units left

Graphing Translations:

Use the graph of the parent function $y = \sin x$ to describe the translations below.

a. $y = \sin x + 3$

- Shift 3↑

b. $y = \sin\left(x - \frac{\pi}{2}\right)$

- Shift $\frac{\pi}{2}$ right

c. $y = \sin(x + \pi) - 2$

- Shift π left & 2 down

Which translation above is a phase shift? b and c

Use the graph of the parent function $y = \cos x$ to describe the transformations below.

a. $y = \cos\left(x + \frac{\pi}{2}\right) - 3$

- Shift $\frac{\pi}{2}$ left and 3 down

b. $y = \cos(x - \pi) + 2$

- Shift π right and 2 up

Key Concepts

Summary — **Families of Sine and Cosine Functions**

Parent Function	Transformed Function
$y = \sin x$	$y = a \sin b(x - h) + k$
$y = \cos x$	$y = a \cos b(x - h) + k$

- $|a|$ = amplitude (vertical stretch or shrink)
- $\frac{2\pi}{b}$ = period (when x is in radians and $b > 0$)
- h = phase shift, or horizontal shift
- k = vertical shift

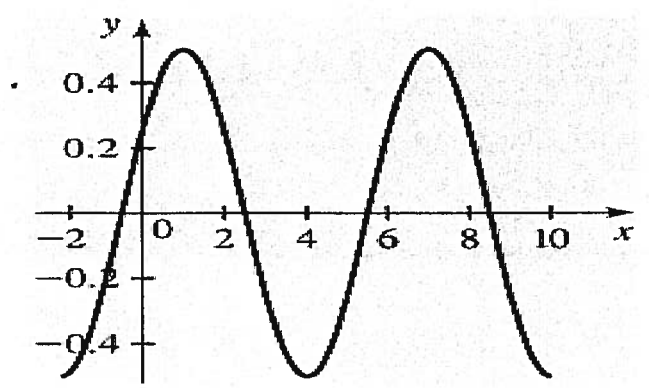
Example: Find the amplitude, period and phase shift of $y = \frac{1}{2}\cos\left(\frac{\pi}{3}(x - 1)\right)$.

We have: Amplitude = |a| = $\frac{1}{2}$

Period = $\frac{2\pi}{b} = \frac{2\pi}{\pi/3} = 6$

Phase shift = h = 1 (shift 1 to the right)

From this information it follows that one period of this cosine curve begins at 1 and ends at (1 + 6 = 7).



a. Find the amplitude, period, phase & vertical shift of $y = \sin\left(2x - \frac{\pi}{2}\right) - 3$.

$\frac{2x}{2} - \frac{\pi}{2}$

$y = \sin 2(x - \pi) - 3$

$x - (\frac{\pi}{2} \cdot \frac{2}{1})$

$a = 1$

$p = \frac{2\pi}{2} = \pi$

phase shift: $\pi = \pi$ units right

vert shift: $-3 = 3$ units down

b. Find the amplitude, period, phase & vertical shift of $y = -3\cos(3x + 2) + 2$.

$a = 3$

$p = \frac{2\pi}{3}$

$y = -3\cos 3(x + \frac{2}{3}) + 2$

phase shift: $\frac{-2}{3} = \frac{2}{3}$ unit left

vert shift: $2 = 2$ units up

Writing a Translation: $y = \sin b(x - h) + k$ or $y = \cos b(x - h) + k$

Write an equation for each translation.

a. $y = \sin x$, π units down

$y = \sin x - \pi$

b. $y = -\cos x$, 2 units to the left

$y = -\cos(x + 2)$

c. $y = \cos x$, $\frac{\pi}{2}$ units up

$y = \cos x + \frac{\pi}{2}$

d. $y = 2\sin x$, $\frac{\pi}{4}$ units to the right

$y = 2\sin(x - \pi/4)$

Practice 13-7 — Translating Sine and Cosine Functions

Find the amplitude, period, phase & vertical shift for each function below.

1. $y = -\sin\left(x + \frac{\pi}{2}\right)$

$a = 1$ $\quad p = 2\pi$ $\quad$ phase shift: $h = -\frac{\pi}{2}$ $\frac{\pi}{2}$ to left

2. $y = \cos\frac{1}{2}x + 1$

$a = 1$ $\quad p = 4\pi$ $\quad$ vert shift: $k = 1$ 1 unit up

3. $y = 3\cos\left(\frac{\pi}{2} - \pi\right)$

$y = \cos\frac{\pi}{2}(x - 2)$

$a = 3$ $\quad p = 4$ $\quad$ phase shift: $h = 2$ 2 units right

4. $y = \sin(3x - \pi)$

$y = \sin 3\left(x - \frac{\pi}{3}\right)$

$a = 1$ $\quad p = \frac{2\pi}{3}$ $\quad$ phase shift: $h = \frac{\pi}{3}$ $\frac{\pi}{3}$ units

5. $y = -2\sin x + 1$

$a = 2$ $\quad p = 2\pi$ $\quad$ vert shift: $k = 1$ 1 unit up

6. $y = -\cos\left(2x + \frac{\pi}{2}\right)$

$y = -\cos 2\left(x + \frac{\pi}{4}\right)$

$a = 1$ $\quad p = \pi$ $\quad$ phase shift: $h = -\frac{\pi}{4}$ $\frac{\pi}{4}$ units left

7. $y = \frac{1}{2}\cos x + 3$

$a = \frac{1}{2}$ $\quad p = 2\pi$ $\quad$ vert shift: $k = 3$ 3 units up

8. $y = \sin\left(\frac{1}{2}x + \frac{\pi}{2}\right)$

$y = \sin\frac{1}{2}(x + \pi)$

$a = 1$ $\quad p = 4\pi$ $\quad$ phase shift: $h = -\pi$ π units left

9. $y = -2\cos x + 3$

$a = 2$ vert shift: $k = 3$

$p = 2\pi$ 3 units up

10. $y = \sin 2x + 1$

$a = 2$ vert shift: $k = 1$

$p = \pi$ 1 unit up

Write a function for each translation.

14. $y = \sin x$, 2 units down

$y = \sin x - 2$

15. $y = \cos x$, π units left

$y = \cos(x + \pi)$

16. $y = \cos x$, $\frac{\pi}{4}$ units up

$y = \cos x + \frac{\pi}{4}$

17. $y = \sin x$, 3.2 units to the right

$y = \sin(x - 3.2)$

Find the amplitude, period, phase & vertical shift for each function below.

18. $y = 3\cos x + 2$

$a = 3$

$p = 2\pi$

VS: $k = 2$

2 units up

19. $y = -2\sin\left(x + \frac{\pi}{2}\right)$

$a = 2$

$p = 2\pi$

PS: $h = -\frac{\pi}{2}$

$\frac{\pi}{2}$ units left

20. $y = \cos 2x + 1$

$a = 1$

$p = \pi$

VS: $k = 1$

1 unit up

21. $y = -\sin\left(x - \frac{\pi}{3}\right)$

$a = 1$

$p = 2\pi$

PS: $h = \pi/3$

$\pi/3$ units right

22. $y = \frac{1}{2}\cos x - 3$

$a = \frac{1}{2}$

$p = 2\pi$

VS: $k = -3$

3 units down

23. $y = \cos\frac{1}{2}x - 2$

$a = 1$

$p = 4\pi$

VS: $k = -2$

2 units down